

TRIGONOMETRIC IDENTITIES (FOR MAC1114)

1)

Angle	0°	$\frac{\pi}{6}=30^\circ$	$\frac{\pi}{4}=45^\circ$	$\frac{\pi}{3}=60^\circ$	$\frac{\pi}{2}=90^\circ$
Sin x	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan x	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef

2) a) $\sin(x + y) = \sin x \cos y + \cos x \sin y$ b) $\sin(x - y) = \sin x \cos y - \cos x \sin y$

c) $\cos(x + y) = \cos x \cos y - \sin x \sin y$ d) $\cos(x - y) = \cos x \cos y + \sin x \sin y$

e) $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$ f) $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

3) a) $\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$

b) $\cos x \sin y = \frac{1}{2} [\sin(x + y) - \sin(x - y)]$

c) $\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$

d) $\cos x \cos y = \frac{1}{2} [\cos(x + y) + \cos(x - y)]$

4) a) $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$ b) $\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$

c) $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$ d) $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$

5) a) $\sin 2x = 2 \sin x \cos x$ b) $\cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 1 - 2 \sin^2 x \\ 2 \cos^2 x - 1 \end{cases}$ c) $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

6) a) $\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$

b) $\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$

c) $\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x}$